DO NOW

Integrate: $\int \frac{1}{5x} dx$

$u = 5x$
$du = 5dx$
$\frac{du}{5} = dx$

$\int \frac{1}{u} \frac{du}{5}$

$\frac{1}{5}\int \frac{1}{u} du$

$\frac{1}{5}\ln|u| + C$

$\boxed{\frac{1}{5}\ln|5x| + C}$

5.7 The Natural Logarithmic Function: Integration

Log Rule for Integration

1. $\int \frac{1}{x} dx = \ln|x| + C$

2. $\int \frac{1}{u} du = \ln|u| + C$

3. $\int \frac{u'}{u} dx = \ln|u| + C$

*Use whenever x^{-1} or when the degree of the numerator < degree of denominator

Examples:

1. $\int \frac{1}{4x} dx$

$u = 4x$
$du = 4dx$

$\frac{1}{4}\int \frac{1}{u} du$

$\frac{1}{4}\ln|u| + C$

$\boxed{\frac{1}{4}\ln|4x| + C}$

2. $\int \frac{1}{3x-1} dx$

$u = 3x-1$
$du = 3dx$

$\frac{1}{3}\int \frac{1}{u} du$

$\frac{1}{3}\ln|u| + C$

$\boxed{\frac{1}{3}\ln|3x-1| + C}$

3. Find the area of $y = \frac{x}{x^2+1}$ bounded by the x-axis and $x = 3$.

$\int_0^3 \frac{x}{x^2+1} dx$

$\frac{1}{2}\int_1^{10} \frac{1}{u} du$

$\left[\frac{1}{2}\ln|u|\right]_1^{10}$

$\frac{1}{2}\ln 10 - \frac{1}{2}\ln 1$

$\frac{1}{2}\ln 10$

$\approx \boxed{1.151}$

OR

$u = x^2+1$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$x=0 \to u=1$
$x=3 \to u=10$

$\frac{1}{2}\int_0^3 \frac{2x}{x^2+1} dx$

$\left[\frac{1}{2}\ln|x^2+1|\right]_0^3$

Pattern Recognition:

4. $\int \frac{3x^2+1}{x^3+x} dx$

$u = x^3+x$
$du = (3x^2+1)dx$

$\boxed{\ln|x^3+x| + C}$

5. $\int \frac{\sec^2 x}{\tan x} dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$\boxed{\ln|\tan x| + C}$

6. $\int \frac{x+1}{x^2+2x} dx$

$u = x^2+2x$
$du = (2x+2)dx$
$du = 2(x+1)dx$

$\frac{1}{2}\int \frac{2(x+1)}{x^2+2x} dx$

$\boxed{\frac{1}{2}\ln|x^2+2x| + C}$

7. $\int \frac{1}{3x+2} dx$

$u = 3x+2$
$du = 3\,dx$

$\frac{1}{3}\int \frac{3}{3x+2} dx$

$\boxed{\frac{1}{3}\ln|3x+2| + C}$

Guidelines for Integration:

1. Learn the basic list of integration formulas.
2. Use *u*-substitution to rewrite the integrand so it conforms to one of the formulas.
3. If *u*-substitution does not work, try altering the integrand. Try trigonometric identities, multiplication/division/addition/subtraction of the same quantity, long division....Be creative.
4. Try using computer software (calculators) to find antiderivatives.

When degree of Numerator ≥ degree of Denominator
***Try Long Division

Example: $\int \frac{3x^3 + x^2 + 4}{3x + 1}\, dx$

$x^2 + \frac{4}{3x+1}$
$3x+1 \overline{)3x^3+x^2+0x+4}$
$-3x^3+x^2$
4

$$\int \left(x^2 + \frac{4}{3x+1}\right) dx$$
$$\int x^2 dx + \int \frac{4}{3x+1}\, dx$$
$$\int x^2 dx + 4\int \frac{1}{3x+1}\, dx \qquad u = 3x+1,\ du = 3dx$$
$$\int x^2 dx + \frac{4}{3}\int \frac{3}{3x+1}\, dx$$
$$\boxed{\frac{1}{3}x^3 + \frac{4}{3}\ln|3x+1| + C}$$

HOMEWORK

pg 358 - 359; 1 - 18